

More About AGM Revision in Description Logics

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August 15, 2010



Outline of topics

1 Belief Dynamics

- Belief Sets
- Operations
- AGM Revision
- AGM Postulates
- Levi identity

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Part I

Introduction to AGM Theory

Belief Set

AGM theory defines operations on *belief sets*:

Definition (Belief Set)

A *belief set* is a set of sentences $K \subseteq \mathcal{L}$ such that $K = Cn(K)$.



Operations on belief sets

There are three main operations that can be applied to belief sets:

- **Expansion:** is used when the agent has to **add** new information to its belief set.
- **Revision:** is used when the agent has to **add** new information to its belief set in a **consistent way**.
- **Contraction:** is used when an agent wants to **remove** sentences from its belief set.



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Example of a revision

Example:

I thought that no Portuguese writer had won the Nobel prize. I couple of months ago I saw in the newspaper: “Nobel-winning author Jose Saramago dies at 87”. Since I believed that Saramago was Portuguese, I had to revise my believes.

To avoid inconsistencies I had to drop my believe about nobel-wining Portuguese writers, or about the nationality of Saramago. I chose the first one.



Formalizing the revision operation

The belief set of agent, whose belief set was K , after contracting α will be represented as $K * \alpha$

Which properties this operation ($K * \alpha$) should satisfy?

Which sets of formulas are candidates for $K * \alpha$?



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AGM postulates

Alchourrón, Gärdenfors and Makinson (AGM) stated that a revision should satisfy the following postulates:

- **success:** $\alpha \in K * \alpha$
- **closure:** $K * \alpha = Cn(K * \alpha)$
- **inclusion:** $K * \alpha \subseteq K + \alpha$
- **vacuity:** If $K + \alpha$ is consistent then $K * \alpha = K + \alpha$
- **extensionality:** If $Cn(\alpha) = Cn(\beta)$ then $K * \alpha = K * \beta$
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Levi identity

The postulates don't tell how to **construct** the contraction.

Given that we know how to construct a contraction (e.g. using partial meet construction) then we can use the Levi identity to construct the revision.

Levi identity

$$K * \alpha = K - \neg\alpha + \alpha$$



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Representation Theorem

Theorem

*If $-$ is a partial meet contraction then $K - \neg\alpha + \alpha$ satisfies all the AGM postulates for revision. Furthermore, if $K * \alpha$ satisfies all the AGM postulates for revision then there is a partial meet contraction $-$ such that $K * \alpha = K - \neg\alpha + \alpha$*



Motivation

The representation theorem for revision holds for logics that satisfies certain properties.

A generic framework for logics

We will consider a logic as a tuple $\langle \mathcal{L}, Cn \rangle$ where:

- \mathcal{L} is the *language* of the logic i.e.: the set of well formed formulas.
- Cn is a *consequence operator* that given a set of formulas return its consequences ($Cn : 2^{\mathcal{L}} \rightarrow 2^{\mathcal{L}}$).

